Accelerating the evolution of simulated convective atmospheres

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1. Introduction.

Modern stellar structure codes such as MESA [1] employ one-dimensional, parameterized models of stellar convection like mixing length theory. While computationally efficient, such an approach does not fully capture the complex nature in which nonlinear convective dynamics interact with and influence the background stellar structure. Previous authors [2-4] sought to incorporate the results of 2- or 3-dimensional convective simulations into 1-dimensional stellar structure models by using implicit timestepping methods to superstep over the convective dynamics. Unfortunately, they found that the convective dynamics determine the largest timesteps which can be stably achieved. As numerical simulations increasingly explore the high-Rayleigh number (Ra), low-diffusivity regime of turbulent stellar convection, numericists face a stiff problem in which nonlinear convective timescales are orders of magnitude smaller than the thermal relaxation timescales over which the atmospheric structure evolves. Coupling resolved convective dynamics with fast 1-dimensional structure models therefore remains a difficult problem.

In this work, we examine a method for accelerating the thermal evolution of convective atmospheres using the simplest possible model: Boussinesq Rayleigh-Benard convection in both 2- and 3-dimensional, cartesian domains. **This method of accelerated evolution (AE) iteratively uses information about system energy fluxes to instantaneously adjust the thermodynamic structure of the convective domain. We compare AE to the standard evolution (SE) of timestepping through a thermal relaxation timescale.** We use the Dedalus [5] pseudospectral toolkit to perform our simulations.

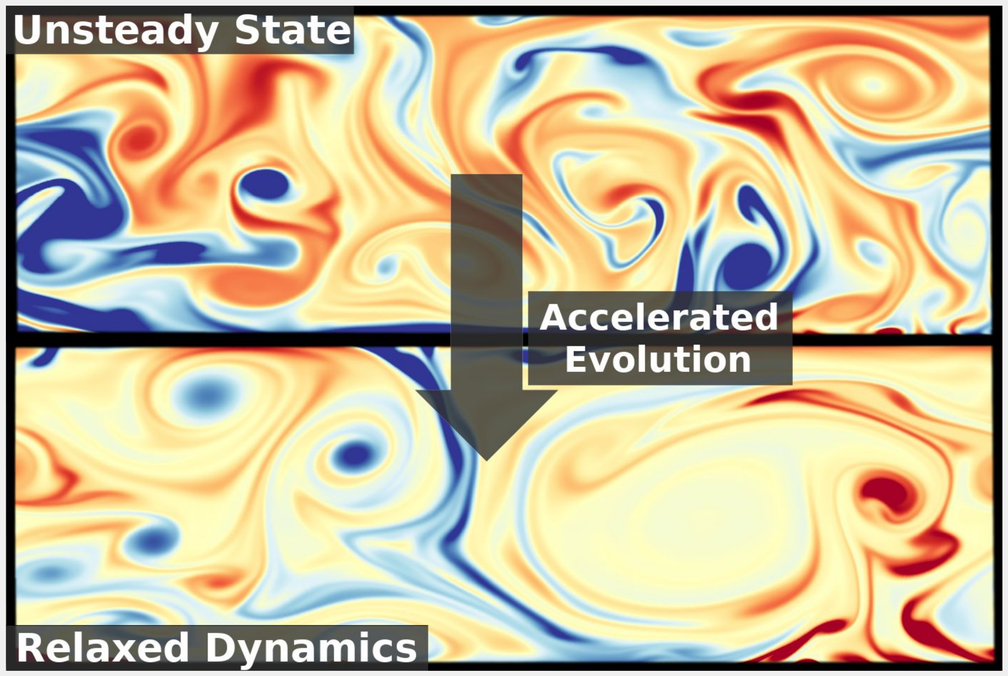

Fig. 1. The temperature anomaly is shown for 2D convection at a Rayleigh number of O(10^{10}). Blue is cold and buoyantly falls, while red is hot and buoyantly rises, and yellow is buoyantly neutral. The top panel is shown in the early, unrelaxed state, in which the convective flows have not clearly organized themselves and the domain is filled with various hot and cold fluid parcels. The bottom panels shows the relaxed state, in which the flows have organized into a clear hot upflow plume at the bottom right and a cold downflow plume near the middle at the top of the domain, with windy regions in between. **The Accelerated Evolution method presented here quickly places solutions whose dynamics and temperature profiles resemble those of the top panel into the relaxed state of the bottom panel, using many fewer cpu-hours than Standard Evolution.**

4. Computational time-savings.

S	nz×nx×ny	N_{CPU}	$t_{CPU,\,SE}$	$t_{CPU,\,AE}$	$t_{CPU,SE}/t_{CPU,AE}$
		2D Runs			
10^2	64×128	32	2.2	4.4	0.5
10^3	128×256	64	53	21	2.56
10^4	256×512	128	1.2×10^3	1.8×10^2	6.67
10^5	512×1024	256	2.4×10^4	2.8×10^3	8.33
		3D Runs			
10^1	32×64×64	512	62	1.1×10^2	0.59
10^2	64×128×128	512	1.9×10^2	1.1×10^2	1.67
10^3	128×256×256	2048	7.0×10^3	1.4×10^3	5.0
10^4	256×512×512	8192	3.3×10^5	2.3×10^4	14.3

Table 1. For select simulations, we show the supercriticality (S ~ Ra/10^3), coefficient resolution (nz, nx, ny), number of CPUs used (N_{CPU}), number of cpu-hours used ($t_{CPU,\,SE/AE}$), and ratio of cpu-hours used in SE compared to AE. **In the astrophysically-interesting, high-supercriticality regime, AE achieves nearly the same solution as SE while using roughly an order of magnitude fewer cpu-hours [6]. For example, the 3D AE case at S = 10^4 is 14.3x faster than the SE case at comparable parameters.**

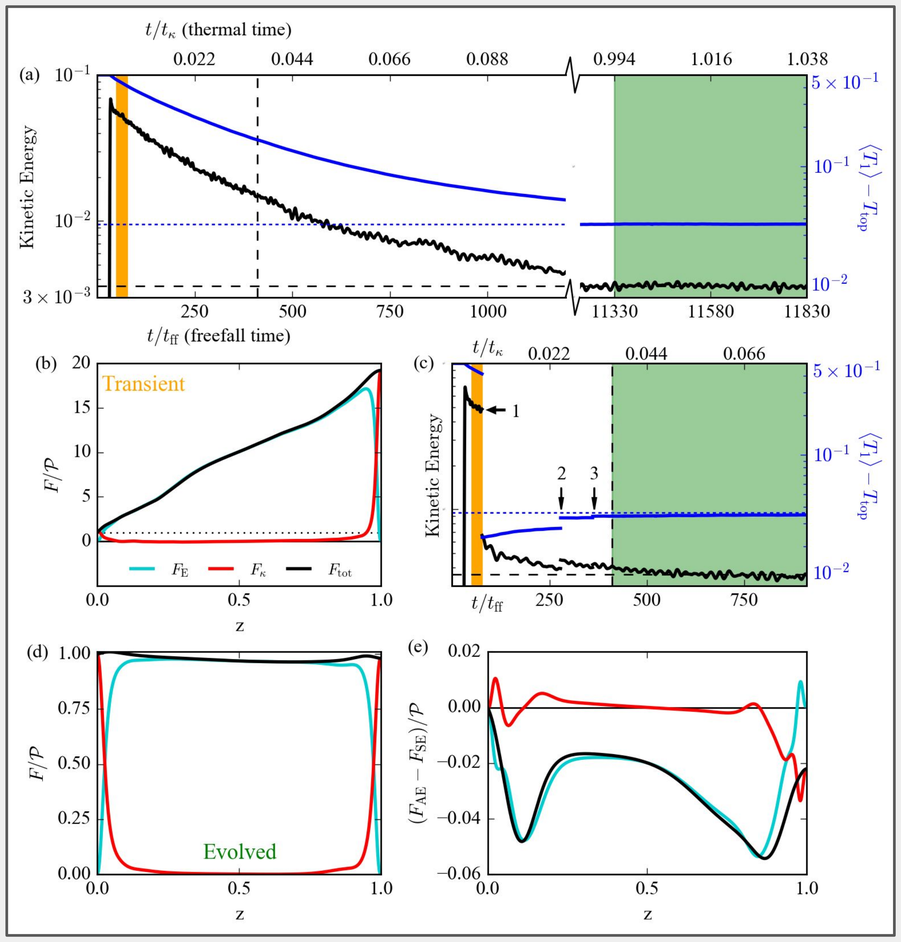

Fig. 2. A comparison of AE and SE, as described in detail in section 2. Reproduced from [6].

2. Method of Accelerated Evolution.

In Fig. 2, we pictorally demonstrate the AE method. **In Fig. 2a, we show traces of volume-averaged kinetic energy (black) and temperature (blue) vs. time for a SE case at Ra ~ 10^8. Thermal relaxation, or the time it takes for the energies to reach their final, equilibrated states (denoted by the horizontal dashed lines) takes thousands of dynamical (freefall) times.** In Fig. 2b, the horizontally-averaged, vertical energy fluxes are shown during the time period highlighted in the orange in Fig. 2a. This period of time is characterized by unsteady dynamics, as in Fig. 1, top panel. Light blue is the convective enthalpy flux (F_E), red is the conductive flux (F_κ), and black is the sum of the two fluxes. The flux leaving the atmosphere at the top (z = 1) is much greater than the flux entering the atmosphere at the bottom (z = 0).

In Fig. 2c, we run the same simulation as in Fig. 2a, but this time we use the AE method three times, as labeled by the three arrows. Rather than running for thousands of dynamical timescales, AE couples the information contained in the fluxes (Fig. 2b) with knowledge about thermal equilibrium in the evolved state to adjust the system's thermal profiles and velocities. **After a few AE iterations, and only a few hundred dynamical timescales of evolution, the system energies are nearly identical to their final values in the SE case in Fig. 2a.** The system is dynamically in a state similar to the bottom panel of Fig. 1 in this state. If we average the system fluxes over 500 dynamical timescales (the green shaded region of Fig. 2c), we see that the system is largely in flux equilibrium (Fig. 2d). When we average SE over the same time window (the green shaded region in Fig. 1a), we find that the AE fluxes are within O(5%) of the SE solution at all heights (Fig. 2e).

3. Thermal vs. dynamical convergence

Not all convective simulations are used to inform theories of stellar structure. Many studies of convection aim to measure the dynamical nature of convection at different points in parameter space. The Nusselt number (Nu) quantifies the efficiency of convective heat transport and has been thoroughly studied in the literature. In Fig. 3, we compare the evolution of Nu (black lines) to the evolution of the thermal energy (blue lines) for the same SE (a) and AE (b) runs that we examined in Fig. 2. Nu is temporally chaotic, and its rolling average is overplotted in dark grey.

While the thermal energy takes thousands of freefall timescales to approach its final value, the mean value of Nu equilibrates within a few hundred freefall timescales. However, Nu's temporal variations away from its mean vary in a vastly different way in the equilibrated and unequilibrated states. **AE is therefore best employed to studies that desire knowledge about more than just the mean values of evolved nondimensional fluid numbers like the Reynolds number or Nu.**

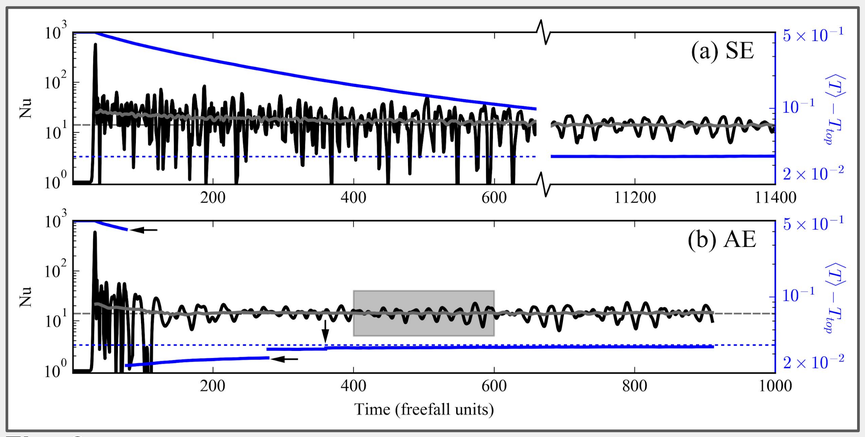

Fig. 3. Time traces of the thermal energy (blue) and Nusselt number (Nu, black) for the AE (top) and SE (bottom) cases depicted in Fig. 2. A rolling time-average of Nu is overplotted in grey for both cases. While the thermal evolution of these systems is very long, the mean value of Nu approaches its final value rapidly. The variations in Nu, particularly in the greyed box in (b), are examined in more detail in [6], particularly in Fig. 3.

5. Conclusions & Extensions.

Here we have briefly described a method of accelerated evolution (AE), and shown how it compares to standard evolution (SE) in a few key measures. Importantly, AE achieves a solution similar to SE using significantly fewer cpu-hours of evolution (Table 1). In Anders, Brown, and Oishi 2018 [6], we show that AE and SE work at many different values of the Rayleigh number, not just in the Ra ~10^8 case presented here.

While the case presented here (Boussinesq Rayleigh-Benard convection) is the simplest possible case, the method used here is generalizable and should be extended to studies of stratified convection [7], internally heated convection [8], or rotating convection [9]. The road between what we present here and using AE to inform stellar evolution models is long, but the similarity between AE and SE and the timesavings achieved by AE suggest that such an application is feasible.

Note: This work can be found online in its published form:
Anders, Brown, & Oishi 2018. "Accelerated evolution of convective simulations." Phys. Rev. Fluids 3, 083502. Published 21 Aug. 2018.

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[References] [1] Paxton et al. 2011 ApJS 192. [2-4] Viallet et al 2011, 2013, 2016 A&A [5] http://dedalus-project.org/ [6] Anders, Brown, & Oishi 2018 PRF 3. [7] Anders & Brown 2017 PRF 2. [8] Goluskin 2016, springer. [9] Anders et al. 2019, in review.